ADVANCED CIRCLE PROBLEMS

TOO MANY SOLUTIONS

Homework

1. Find the equation of the circle with center at the origin and the given **diameter**:

   a. $d = 10$ b. $d = 6\sqrt{5}$ c. $d = 3\sqrt{2}$
   d. $d = 7$ e. $d = 5\sqrt{3}$ f. $d = 0$

2. The ***unit circle*** is the circle whose center is at the origin and whose radius is 1.

   a. Find the equation of the unit circle.
   b. What is the area of the unit circle? [$A = \pi r^2$]
   c. What is the circumference of the unit circle? [$C = 2\pi r$]

3. a. Find two horizontal tangent lines to the circle $x^2 + y^2 = 16$.
   b. Find two vertical tangent lines to the circle $x^2 + y^2 = 49$.

4. Find the midpoint of the segment connecting the given pair of points:

   a. (2, 7) and (6, 13) b. (–8, 5) and (0, 11)
   c. (0, 0) and (–22, 9) d. (3, –4) and (–8, 9)

EXAMPLE 3: **Find the equation of the circle whose center is (9, –12) and whose diameter is $8\sqrt{5}$.**

Solution: We're given the diameter instead of the radius, but there's no need to worry. Since the radius is always half the diameter, we can calculate the radius:

$$r = \frac{d}{2} = \frac{8\sqrt{5}}{2} = \frac{\cancel{2}\cdot 4\sqrt{5}}{\cancel{2}} = 4\sqrt{5}$$

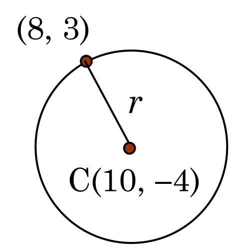
When we know the radius, recall that it has to be squared before we can put it into the formula: $\left(4\sqrt{5}\right)^2 = 4^2\cdot\sqrt{5}^{\,2} = 16\cdot 5 = 80$.

Using the center given in the problem, the equation of the circle becomes

$$\boxed{(x-9)^2+(y+12)^2 = 80}$$

EXAMPLE 4: **Find the equation of the circle given that its center is at the point (10, –4) and that it passes through the point (8, 3).**

Solution: We're given the center, so that's no problem. However, we don't have the radius — we don't even have the diameter. Can we calculate the radius from the picture at the right?



We know that the point (8, 3) is on the circle, and remember that the radius of a circle is the distance from the center of the circle to any point on the circle. So, we can figure out the radius merely by calculating the distance from the center (10, –4) to the point (8, 3). The Distance Formula rescues us again:

$$r = \sqrt{(\Delta x)^2+(\Delta y)^2}$$

$$\Rightarrow \quad r = \sqrt{(10-8)^2 + (-4-3)^2}$$

$$\Rightarrow \quad r = \sqrt{2^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$$

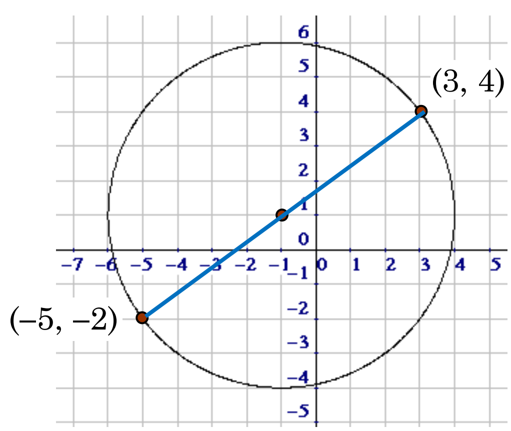
It's time to recap what we have so far: First, the center (10, –4) was given to us; second, we have just calculated the radius to be $\sqrt{53}$. Placing these values into the standard form of a circle gives

$(x-10)^2 + (y+4)^2 = \left(\sqrt{53}\right)^2$; the final answer is

$$\boxed{(x-10)^2 + (y+4)^2 = 53}$$

<u>EXAMPLE 5:</u> **Find the equation of the circle given that the endpoints of a diameter are the points (–5, –2) and (3, 4).**

<u>Solution:</u> As you know by now, the definition of a circle requires that we know the center and the radius. This problem gives us neither of these. Let's draw a picture and see what we can see.



Look at the diameter with endpoints (–5, –2) and (3, 4). Is it clear that the center must be <u>halfway</u> between the endpoints? What have you learned that will give us the point that is midway between two other points? The Midpoint Formula is just perfect. So the center must be

$$\left(\frac{-5+3}{2}, \frac{-2+4}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$$

Now look back at the picture again. As in the previous example, the radius of the circle is the distance between the center (–1, 1) and either one of the diameter endpoints. Let's use the endpoint (3, 4). Then,

$$r = \sqrt{[3-(-1)]^2+(4-1)^2} = \sqrt{(3+1)^2+(4-1)^2}$$
$$= \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Finally, with a center of (–1, 1) and a radius of 5, the equation of the circle must be

$$\boxed{(x+1)^2+(y-1)^2 = 25}$$

Homework

5. Find the equation of the circle with the given center and diameter:

   a. C(2, 3) $d = 12$ b. C(–4, 1) $d = 3$

   c. C(–1, –5) $d = 2\sqrt{7}$ d. C(7, –3) $d = 5\sqrt{2}$

6. In each problem below, the center C of a circle is given, and a point P on the circle is given. Find the equation of the circle.

   a. C(2, 5) P(3, 7) b. C(–1, 4) P(–2, 5)

   c. C(0, –3) P(1, –1) d. C(7, 0) P(7, –5)

   e. C(–3, 6) P(–2, –3) f. C(7, –2) P(–2, –2)

7. Each pair of points below represents the endpoints of a diameter of a given circle. Find the equation of the circle.

   a. (2, 5) (4, –1) b. (–3, –2) (–1, 8)

   c. (5, 0) (7, –4) d. (0, –3) (0, –7)

e. (2, –1) (6, –3) f. (–3, 8) (–1, –2)

❑ SEMICIRCLES

EXAMPLE 6: **Find the equation of the top-half of the circle $x^2 + y^2 = 10$.**

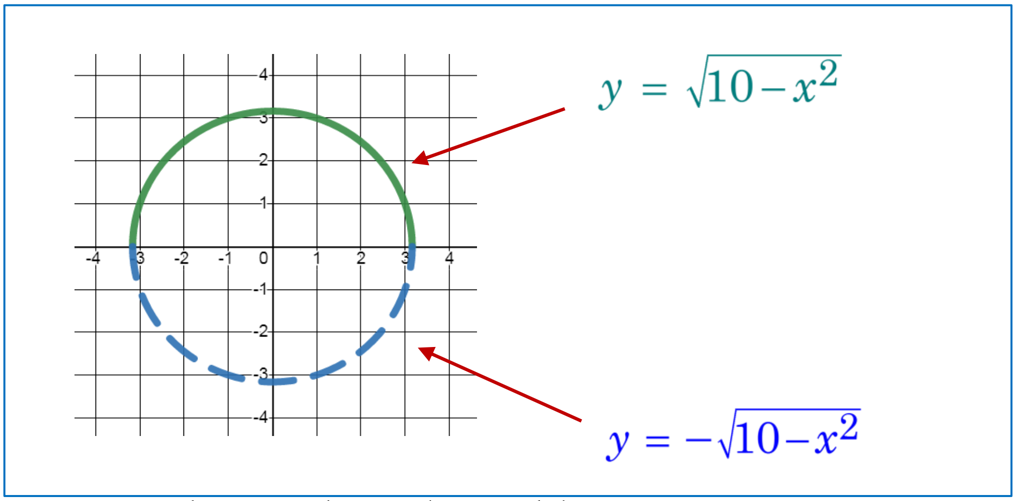
Solution: First, we notice that the given circle has its center at the origin and has a radius of $\sqrt{10}$. If you picture the graph of this circle, you will see that the top half of the circle can be described as that part of the circle which lies in Quadrants I and II (touching the x-axis). Equivalently, this is where the y-coordinate of any point on the circle is never negative, which can be written $y \ge 0$.

Second, we solve the circle equation for y:

$$x^2 + y^2 = 10$$
$$\Rightarrow \quad y^2 = 10 - x^2$$
$$\Rightarrow \quad y = \pm\sqrt{10 - x^2}$$

Since we require that the y-value never be negative, we will take a "subset" to ensure that $y \ge 0$. To do this, simply take the non-negative square root — that is, remove the ± sign.

$$\boxed{y = \sqrt{10 - x^2}}$$



Homework

8. a. Graph $y = \sqrt{25 - x^2}$ b. Graph $y = -\sqrt{25 - x^2}$

   c. T/F: The graphs are disjoint (have no points in common).

9. Describe the graph of $y = \sqrt{20 - x^2}$.

10. Describe the graph of $y = -\sqrt{84 - x^2}$.

11. Find the equation of the bottom half of the circle $x^2 + y^2 = 100$.

12. Find the equation of the top half of the circle $x^2 + y^2 = 30$.

Review Problems

13. Find the equation of the circle given that the endpoints of a diameter are (2, –8) and (8, 0).

14. Find two horizontal tangent lines to the circle in the preceding problem.

15. Find the equation of the top half of the circle with center at the origin and radius 5.

16. Find the equation of the bottom half of the circle with center at the origin and radius 12.

17. Find the center and radius of the circle $x^2 + y^2 - 10x + 2y + 3 = 0$.

18. Find the equation of the circle whose center is $(-1, -3)$ and whose diameter is $12\sqrt{11}$.

19. Assume that the center of a circle is $(9, -3)$, and also assume that the point $(7, -8)$ is on the circle. Find the equation of the circle.

20. In terms of exponents and coefficients, explain why each of the following is <u>not</u> a circle.

a. $2x + 3y = 1$ b. $y = x^2$ c. $y = x^3$

d. $x^2 + 3y^2 = 4$ e. $x^2 - y^2 = 9$ f. $y = \sqrt{x}$

21. True/False:

a. Every circle has at least one intercept.

b. The radius of the circle $x^2 + y^2 = 1$ is 1.

c. A semicircle can be a function.

d. A semicircle must be a function.

e. The derivation of the circle formula is based upon the Distance Formula.

f. $x^2 + y^2 + 4 = 0$ is a circle.

g. $x^2 + y^2 = 1$ is called the unit circle.

h. The center of the circle $(x - 2)^2 + (y - 5)^2 = 10$ is $(-2, -5)$.

i. The radius of the circle $x^2 + y^2 + 8x - 6y + 9 = 0$ is 4.

j. The radius of a circle whose diameter is $25\sqrt{2}$ is $5\sqrt{2}$.

k. The graph of $y = \sqrt{19 - x^2}$ is the top half of a circle.

l. The radius of the above circle is 19.

m. The graph of $10x^2 + 10y^2 = 37$ is a circle.

n. The graph of $10x^2 - 10y^2 = 37$ is a circle.

o. The graph of $10x^2 + 9y^2 = 37$ is a circle.

p. The area of the circle $x^2 + y^2 = 25$ is 25π.

Solutions

1. a. 10 b. 13 c. 25 d. $3\sqrt{2}$ e. $5\sqrt{2}$
f. $\sqrt{58}$ g. $9\sqrt{2}$ h. 0 i. 1 j. $\sqrt{185}$

2. r represents the radius (which is a distance), and thus makes sense only if $r > 0$ [a circle with zero or negative radius wouldn't be a circle].

3. It must be 18, since the distance from any point on the circle to the center must be the radius, which is 18.

4. a. C(0, 0) $r = 5$ b. C(0, 0) $r = 12$ c. C(0, 0) $r = 1$
d. C(0, 0) $r = \sqrt{17}$ e. C(0, 0) $r = 3\sqrt{3}$ f. C(0, 0) $r = 10\sqrt{2}$
g. It's not a circle; the graph is just the origin.
h. It's not a circle; there are two reasons. First, the radius would be $\sqrt{-9}$,which is not a real number. Second, if two numbers are squared and then added together, there's no way that sum can be negative.
i. It's a line, not a circle.
j. It's not a circle -- what is it?

5. a. $x^2 + y^2 = 100$ b. $x^2 + y^2 = 625$ c. $x^2 + y^2 = 1$
d. $x^2 + y^2 = 256$ e. $x^2 + y^2 = 11$ f. $x^2 + y^2 = 18$
g. $x^2 + y^2 = 80$ h. $x^2 + y^2 = 63$

6. a. $x^2 + y^2 = 25$ b. $x^2 + y^2 = 45$ c. $x^2 + y^2 = \frac{9}{2}$
d. $x^2 + y^2 = \frac{49}{4}$ e. $x^2 + y^2 = \frac{75}{4}$ f. Not a circle

7. a. $x^2 + y^2 = 1$ b. $A = \pi r^2 = \pi(1^2) = \pi(1) = \pi$
c. $C = 2\pi r = 2\pi(1) = 2\pi$

8. a. $y = 4$ and $y = -4$ b. $x = 7$ and $x = -7$

9. a. C(0, 0) $r = 8$ b. C(0, 0) $r = 2\sqrt{6}$
c. C(0, 7) $r = 2$ d. C(3, 0) $r = \sqrt{5}$
e. C(–1, –8) $r = 2\sqrt{15}$ f. C(2, 3) $r = 3\sqrt{11}$
g. C(–5, 3) $r = 12$ h. C(1, –11) $r = 4\sqrt{3}$
i. It's a line j. It's a parabola

10. a. $x^2 + y^2 = 49$ b. $x^2 + y^2 = 10$
c. $(x-3)^2 + y^2 = 9$ d. $x^2 + (y-4)^2 = 1$
e. $x^2 + (y+2)^2 = 3$ f. $(x+12)^2 + y^2 = 144$
g. $(x-2)^2 + (y-7)^2 = 100$ h. $(x+1)^2 + (y+3)^2 = 12$
i. $(x-3)^2 + (y+4)^2 = 45$ j. $(x+2)^2 + (y-9)^2 = 175$
k. Just the point (1, 2) l. Not a circle; no graph at all

11. a. (4, 10) b. (–4, 8) c. (–11, 9/2) d. (–5/2, 5/2)

12. a. C(–4, 5) $r = 9$ b. C(8, –4) $r = 7$
c. C(–6, –7) $r = 6$ d. C(–7, 0) $r = 5$
e. C(0, 3) $r = 15$ f. C(–4, –3) $r = \sqrt{15}$
g. C(1, 2) $r = 2\sqrt{2}$ h. C(–2, 5) $r = \sqrt{17}$
i. C(10, –3) $r = 2\sqrt{2}$ j. Not a circle

13. a. $(x-2)^2 + (y-3)^2 = 36$ b. $(x+4)^2 + (y-1)^2 = \frac{9}{4}$
c. $(x+1)^2 + (y+5)^2 = 7$ d. $(x-7)^2 + (y+3)^2 = \frac{25}{2}$

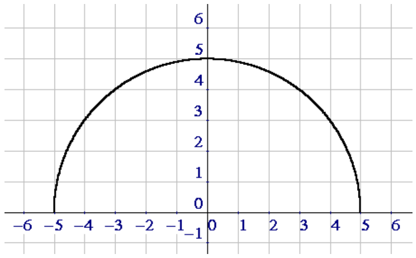
14. a. $(x-2)^2 + (y-5)^2 = 5$ b. $(x+1)^2 + (y-4)^2 = 2$
c. $x^2 + (y+3)^2 = 5$ d. $(x-7)^2 + y^2 = 25$
e. $(x+3)^2 + (y-6)^2 = 82$ f. $(x-7)^2 + (y+2)^2 = 81$

15. a. $(x-3)^2 + (y-2)^2 = 10$ b. $(x+2)^2 + (y-3)^2 = 26$

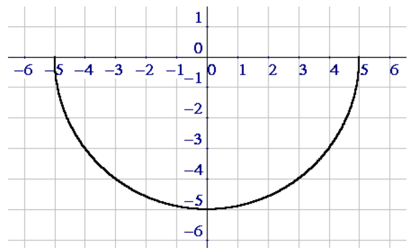
c. $(x-6)^2 + (y+2)^2 = 5$ d. $x^2 + (y+5)^2 = 4$

e. $(x-4)^2 + (y+2)^2 = 5$ f. $(x+2)^2 + (y-3)^2 = 26$

16. a.



b.



c. False; they intersect at the points (5, 0) and (–5, 0).

17. The top-half of a circle with center at the origin and radius $2\sqrt{5}$.

18. The bottom-half of a circle with center at the origin and radius $2\sqrt{21}$.

19. $y = -\sqrt{100-x^2}$

20. $y = \sqrt{30-x^2}$

21. A circle is the set of all points in the plane that are equidistant from a fixed point in the plane.

22. If $k > 0$, the graph is a circle with center at the origin and radius $\sqrt{k}$.
If $k = 0$, the graph is just the single point (0, 0); i.e., the origin.
If $k < 0$, the graph is empty (there's no graph at all).

23. $C(0, 0)$; $r = 2\sqrt{5}$

24. $(x-5)^2 + (y+4)^2 = 25$

25. $y = 1$ and $y = -9$

26. $y = \sqrt{25-x^2}$

27. $y = -\sqrt{144-x^2}$

28. $C(5, -1)$; $r = \sqrt{23}$

29. $(x+1)^2 + (y+3)^2 = 396$

30. $(x-9)^2 + (y+3)^2 = 29$

31. a. $2x + 3y = 1$: At the very least, a circle has both variables squared, which the given equation certainly does not have. In fact, the given equation is a line.

b. $y = x^2$: Again, at the very least both variables need to be squared in order to be a circle. Indeed, the equation is a parabola.

c. $y = x^3$: This is a cubic function, whose graph was discussed in Chapter 20.

d. $x^2 + 3y^2 = 4$: This equation does have both variables squared, but the coefficients of the squared terms (the 1 and the 3) are not the same, so there's no way we could put the equation into the standard form of the circle: $(x - h)^2 + (y - k)^2 = r^2$. The next chapter will show you what the given equation actually looks like.

e. $x^2 - y^2 = 9$: This equation strongly resembles the equation of a circle, but the minus sign between the squared terms kills the deal. Remember that the equation of a circle ultimately comes from the Distance Formula, which has a plus sign between the squared parts. You might see this equation later in your studies.

f. $y = \sqrt{x}$: Certainly, the exponents aren't right for this to be a circle, since the y has an exponent of 1 and the x has an exponent of 1/2. In fact, the graph of this function is the top half of the parabola opening to the right.

32.

a. F	b. T	c. T	d. F	e. T	f. F	g. T	h. F
i. T	j. F	k. T	l. F	m. T	n. F	o. F	p. T

“Education is the vaccine for violence.”

—Edward James Olmos